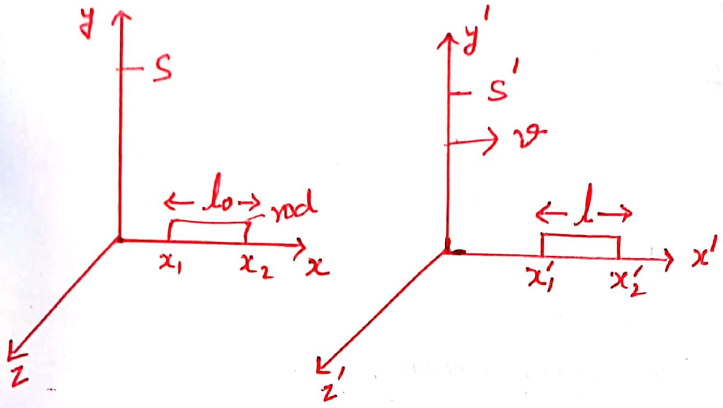
Consequences of Lorentz Transformation

① Length contraction :-



Consider a rod laying at rest along x-axis in S frame, so proper length of the rod $l_0 = x_2 - x_1$ —(I)

Now the measurement in S' frame which is going in positive direction from S with velocity v;

observe length $l = x_2' - x_1'$ —(II)

We know that by Lorentz inverse transformation

$$x_1 = \frac{x_1' + vt'}{\sqrt{1 - v^2/c^2}} \quad , \quad x_2 = \frac{x_2' + vt'}{\sqrt{1 - v^2/c^2}}$$

So

$$x_2 - x_1 = \frac{x_2' + vt' - x_1' - vt'}{\sqrt{1 - v^2/c^2}}$$

$$x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{l_0 = \gamma l}$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$\gamma > 1.$

Therefore, $l_0 > l$

but in perpendicular directions $y' = y$, $z' = z$ No change in length

Thus the length of the rod in all other frame of reference in uniform motion with respect to the frame in which the rod is at rest is shorter than its proper length.

Ex A sphere will look like as a spheroid due to decrease in its diameter parallel to x-axis.